

THIRD SOUTHWESTERN GERMAN WORKSHOP ON GRAPH THEORY - ABSTRACTS

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1. INVITED TALKS

JÓZSEF BALOGH - ON ROBUSTNESS OF THE ERDŐS–KO–RADO THEOREM

A family of subsets of $[n]$ is intersecting if every pair of its sets intersects. Determining the structure of large intersecting families is a central problem in extremal combinatorics, starting with the well-known Erdős–Ko–Rado Theorem. We consider two extensions of it:

Counting variant: Frankl–Kupavskii and Balogh–Das–Liu–Sharifzadeh–Tran showed that for $n \geq 2k + c(k \ln k)^{1/2}$, almost all k -uniform intersecting families are stars. Improving their result, we show that the same conclusion holds for $n \geq 2k + 100 \ln k$.

Random variant: For positive integers n and k with $n \geq 2k + 1$, the Kneser graph $K(n, k)$ is the graph with vertex set consisting of all k -sets of $\{1, \dots, n\}$, where two k -sets are adjacent exactly when they are disjoint. Let $K_p(n, k)$ be a random spanning subgraph of $K(n, k)$ where each edge is included independently with probability p . Bollobas, Narayanan, and Raigorodskii asked for what p does $K_p(n, k)$ have the same independence number as $K(n, k)$

with high probability. Building on work of Das and Tran and of Devlin and Kahn, we resolve this question.

Our proofs uses, among others, the graph container method and the Das–Tran removal lemma.

It is joint work with Lina Li, Ramon Garcia, Adam Wagner; and with Robert Krueger and Haoran Luo.

STEFAN GLOCK - THE n -QUEENS COMPLETION PROBLEM

An n -queens configuration is a placement of n mutually non-attacking queens on an $n \times n$ chessboard. The n -queens completion problem, introduced by Nauck in 1850, is to decide whether a given partial configuration can be completed to an n -queens configuration. Since the n -queens completion problem has been used for decades in Artificial Intelligence papers as a benchmark problem, it is important to understand how hard it actually is. Perhaps surprisingly, we show that any placement of at most $n/60$ mutually non-attacking queens can always be completed. We also provide partial configurations of roughly $n/4$ queens that cannot be completed. Some interesting problems remain open for future research. In the talk, I will discuss our results and our main tools, which include a reduction to rainbow matchings in bipartite graphs, probabilistic arguments and linear programming duality. This is joint work with David Munhá Correia and Benny Sudakov.

ADRIANA HANSBERG - UNAVOIDABLE PATTERNS IN 2-COLORINGS OF THE COMPLETE GRAPH

Ramsey’s theorem states that, for any integer t , if n is a sufficiently large integer, then every 2-coloring of the edges of a complete graph on n vertices K_n contains a monochromatic complete graph of order t . On the other hand, Turán’s theorem says that, if a graph has sufficiently many edges, then it contains a complete graph on t vertices as a subgraph. In relation to these two types of problems, we study which 2-colored patterns are forced to appear in differently saturated 2-colorings of the edges of a complete graph on n vertices, K_n provided n is large enough. This is a joint work with Yair Caro and Amanda Montejano.

MATTHIAS KRIESELL - GRAPH DECOMPOSITION WITH CONSTRAINTS TO MINIMUM DEGREE AND CONNECTIVITY

CARSTEN THOMASSEN conjectured in 1983 that every graph G of minimum degree $\delta(G) \geq s + t + 1$ admits a partition (A, B) of $V(G)$ such that $\delta(G[A]) \geq s$ and $\delta(G[B]) \geq t$. Whereas this has been answered affirmatively by MICHAEL STIEBITZ in 1996, the connectivity version, where δ is replaced with the vertex-connectivity κ at all three places, remains open. In the talk I will consider related problems in the area, among them (i) a recent minimum degree version of a yet unsolved conjecture of WILLIAM MCCUAIG and KATSUHIRO OTA that for given ℓ , every sufficiently large 3-connected graph admits a partition (A, B) of $V(G)$

such that $G[A]$ is 1-connected, $G[B]$ is 2-connected, and $|A| = \ell$, and (ii) some results and conjectures on planar 5-connected graphs.

MATTHEW KWAN - HIGH-GIRTH STEINER TRIPLE SYSTEMS

In this talk I'll discuss our recent resolution of a conjecture due to Erdős on the existence of Steiner triple systems with arbitrarily high girth. I'll start by giving a brief overview of random greedy processes and the method of iterative absorption, and discussing the relevant challenges in the high-girth setting. Then, I'll outline some ideas that help overcome these challenges, related to sparsification, efficient absorption, and “retrospective” analysis of random processes. This is joint work with Ashwin Sah, Mehtaab Sawhney, and Michael Simkin.

DHRUV MUBAYI - RECENT ADVANCES IN RAMSEY THEORY

We describe recent progress on classical problems in graph and hypergraph Ramsey theory. We will show an approach which has the potential to determine the polynomial growth rate of all classical off-diagonal graph Ramsey numbers by using pseudo-randomness. For hypergraphs, we will describe the state of the art for classical off-diagonal Ramsey numbers. Finally, we will discuss a solution to a 1972 question of Erdős and Hajnal on a generalized Ramsey problem for which Erdős offered a \$500 prize. This is joint work with Jacques Verstraete, Andrew Suk, and Alexander Razborov.

OLEG PIKHURKO - TURAN-EXTREMAL HYPERGRAPHS CONSTRUCTED FROM MIXTURE OF PATTERNS

Previously, it was shown in [O.Pikhurko “On Possible Turan Densities”, Israel J Math, 201 (2014) 415-454] that if one starts with a pattern P and takes its recursive blow-ups then the maximum k -graphs obtainable this way are exactly the maximum F -free k -graphs for some finite k -graph family F . I will discuss an on-going work with Xizhi Liu where we extend this result to constructions where different patterns can be mixed, and present some applications.

MATHIAS SCHACHT - EXTREMAL PROBLEMS IN HYPERGRAPHS WITH QUASIRANDOM LINKS

Extremal problems for 3-uniform hypergraphs concern the maximum cardinality of a set E of 3-element subsets of a given n -element set V such that for any ℓ elements of V at least one triple is missing in E . This innocent looking problem is still open, despite a great deal of effort over the last 80 years. We consider a variant of the problem by imposing additional restrictions on the distribution of the 3-element subsets in E , which are motivated by the theory of *quasirandom hypergraphs*. These additional assumptions yield a finer control over the corresponding extremal problem. In fact, this leads to many interesting and more

manageable subproblems, some of which were already considered by Erdős and Sós in the 1980s. In this talk we consider hypergraphs whose vertices have quasirandom link graphs and report on recent progress for the corresponding extremal problems based on joint work with S. Berger, S. Piga, Chr. Reiher, and V. Rödl.

KATHERINE STADEN - THE ERDŐS-ROTHSCHILD PROBLEM

Questions about forbidden subgraphs form a central part of extremal graph theory. In this talk I will discuss a colourful problem of this sort: the Erdős-Rothschild problem from 1974. Consider an n -vertex graph G whose edges are coloured with s colours so that there is no monochromatic clique of size k , and call such a colouring of G valid. The problem is to determine the maximum number of valid colourings over all n -vertex graphs G . It is in general wide open and an exact (or even asymptotic) answer is only known for a few pairs (k, s) . In this talk I will discuss new exact results, and intriguing connections to algebra and designs. Joint work with Oleg Pikhurko.

GÁBOR TARDOS - EDGE-ORDERED GRAPHS WITH ALMOST LINEAR EXTREMAL FUNCTIONS

Turán type extremal theory of graphs has a long history and can boast with a wide array of applications. Here I will speak of an extension of this theory to edge-ordered graphs: simple graphs with a linear order on their edges. In their extremal theory we ask how many edges an edge-ordered graph may have on a given number n of vertices if it does not contain a “forbidden pattern” with a *edge order*: this is the extremal function of the forbidden pattern. In a paper with Gerbner, Methuku, Nagy, Pálvölgyi, and Vizer we started to develop this extremal theory.

The classical (unordered) extremal theory has a simple dichotomy: if the forbidden pattern is a forest, then the extremal function is linear, otherwise it is $\Omega(n^c)$ for some $c > 1$. While such a clean dichotomy fails for vertex- or edge-ordered graphs a similar dichotomy was conjectured for both cases. The conjecture is still open for vertex-ordered graphs with several partial results already achieved. In the talk I will focus on our recent solution for the edge-ordered dichotomy conjecture with Gaurav Kucheriya.

LUTZ WARNKE - THE DEGREE-RESTRICTED RANDOM PROCESS IS FAR FROM UNIFORM

The random d -process corresponds to a natural algorithmic model for generating d -regular graphs: starting with an empty graph on n vertices, it evolves by sequentially adding new random edges so that the maximum degree remains at most d . In 1999 Wormald conjectured that the final graph of the random d -process is “similar” to a uniform random d -regular graph.

We show that this conjecture does not extend to a natural generalization of this process with mixed degree restrictions, i.e., where each vertex has its own degree restriction (under some

mild technical assumptions). Our proof uses the method of switchings, which is usually only applied to uniform random graph models – rather than to stochastic processes.

Based on joint work in progress with Mike Molloy and Erlang Surya.

DAVID WOOD - PRODUCT STRUCTURE OF GRAPH CLASSES WITH BOUNDED TREewidth

Graph product structure theory describes complicated graphs as subgraphs of strong products of simpler building blocks. The building blocks typically have bounded treewidth, which is the standard measure of how similar a graph is to a tree. Recent results show that numerous graph classes with unbounded treewidth can be described this way. This work shows that graph product structure theory is relevant even for graphs with bounded treewidth. We show that certain graphs with bounded treewidth can be described as subgraphs of the strong product of a graph with smaller treewidth and a bounded-size complete graph. To this end, define the *underlying treewidth* of a graph class \mathcal{G} to be the minimum non-negative integer c such that, for some function f , for every graph $G \in \mathcal{G}$ there is a graph H with $\text{tw}(H) \leq c$ such that G is isomorphic to a subgraph of $H \boxtimes K_{f(\text{tw}(G))}$. We introduce disjointed coverings of graphs and show they determine the underlying treewidth of any graph class. Using this result, we prove that the class of planar graphs has underlying treewidth 3; the class of $K_{s,t}$ -minor-free graphs has underlying treewidth s (for $t \geq \max\{s, 3\}$); and the class of K_t -minor-free graphs has underlying treewidth $t - 2$. In general, we prove that a monotone class has bounded underlying treewidth if and only if it excludes some fixed topological minor. This is joint work with Rutger Campbell, Katie Clinch, Marc Distel, J. Pascal Gollin, Kevin Hendrey, Robert Hickingbotham, Tony Huynh, Freddie Illingworth, Youri Tamitegama, and Jane Tan.

2. CONTRIBUTED TALKS

FABRICIO BENEVIDES - ON HEILBRONN TRIANGLE-TYPE PROBLEMS IN HIGHER DIMENSIONS

The Heilbronn triangle problem is a classical geometrical problem that asks for a placement of n points in the unit-square $[0, 1]^2$, that maximizes the smallest area of a triangle formed by those points. This problem has natural generalizations to higher dimensions. For integers $k, d > 1$ and a set P of n points in $[0, 1]^d$, let s be the minimum of $(k - 1)$ and d ; and let $V(k, d, P)$ be the minimum s -dimensional volume of the convex hull of k points in P . Here, instead of considering the supremum of $V(k, d, P)$, we consider its average value, $\text{avgDelta}(k, d, n)$, when the n points in P are chosen independently and uniformly at random in $[0, 1]^d$. We prove that $\text{avgDelta}(k, d, n)$ is of order n to the power of $(-k/1 + |d - k + 1|)$, for every fixed $k, d > 1$.

FELIX CLEMEN - THE SPECTRUM OF TRIANGLE-FREE GRAPHS

We prove a conjecture by Brandt from 1997 on the spectrum of triangle-free graphs: Given an n -vertex graph G , let $\lambda_n \leq \dots \leq \lambda_1$ be the eigenvalues of the adjacency matrix of G . Every regular triangle-free n -vertex graph G satisfies $\lambda_1 + \lambda_n \leq 4n/25$. This is a subproblem of two famous conjectures by Erdős.

- (1) Sparse-Half-Conjecture: Every n -vertex triangle-free graph has a subset of vertices of size $n/2$ spanning at most $n^2/50$ edges.
- (2) Every n -vertex triangle-free graph can be made bipartite by removing at most $n^2/25$ edges.

This is joint work with József Balogh, Bernard Lidický, Sergey Norin and Jan Volec.

ANN-KATHRIN ELM - RADIAL PATH WIDTH

Usually, the width of a tree decomposition of a graph is measured in terms of the size of the parts of the tree decomposition, and size means number of vertices. The grid theorem states that a graph either has a tree decomposition of low width or a large grid as a minor. The radial width of a tree decomposition is computed similarly to the usual width of the tree decomposition, only now the size of a part means its radius. There is no similar statement to the grid theorem known for radial tree width yet, but there is for the case where the decomposition trees are paths.

This is joint work with Sandra Albrechtsen, Reinhard Diestel, Eva Fluck, Raphael W. Jacobs, Paul Knappe and Paul Wollan

JOSHUA ERDE - EXPANSION IN THE GIANT COMPONENT OF THE PERCOLATED HYPERCUBE

As in the case of the binomial random graph, it is known that the behaviour of a random subgraph of a d -dimensional hypercube, where we include each edge independently with probability p , undergoes a phase transition when p is around $1/d$. More precisely, answering a question of Erdős and Spencer, it was shown by Ajtai, Komlós and Szemerédi that significantly above this value of p , in the supercritical regime, whp this random subgraph has a unique ‘giant’ component, whose order is linear in the order of the hypercube.

In the binomial random graph much more is known about the complex structure of this giant component, much which can be deduced from more recent results about the likely expansion properties of the giant component. We show that whp the giant component L of a supercritical random subgraph of the d -dimensional hypercube has reasonably good expansion properties, and use this to deduce some structural information about L . In particular this leads to polynomial (in d) bounds on the diameter of L and the mixing time of a random walk on L , answering questions of Pete, and of Bollobás, Kohayakawa, and Łuczak.

Joint with Mihyun Kang and Michael Krivelevich.

ALBERTO ESPUNY DÍAZ - POWERS OF HAMILTON CYCLES IN GRAPHS PERTURBED BY
A RANDOM GEOMETRIC GRAPH

A random geometric graph $G^d(n, r)$ is obtained by placing n vertices uniformly and independently at random in the hypercube $[0, 1]^d$ and joining two vertices by an edge if the distance between them is at most r . We study the problem of the containment of k -th powers of Hamilton cycles in the union of $G^d(n, r)$ with an n -vertex graph H_n with minimum degree αn . For all values of k , d and α , we provide asymptotically optimal values for r which ensure the union contains the k -th power of a Hamilton cycle with high probability. Our result implies asymptotically optimal conditions for the containment of other spanning structures.

This is joint work with Joseph Hyde.

VICTOR FALGAS-RAVRY - TOWARDS A 1-DEPENDENT VERSION OF THE
HARRIS–KESTEN THEOREM

Consider a random subgraph of the square integer lattice \mathbb{Z}^2 obtained by including each edge independently at random with probability p , and leaving it out otherwise. The Harris–Kesten theorem states that if p is at most $1/2$, then almost surely all connected components in this random subgraph are finite, while if $p > 1/2$ then almost surely there exists a unique infinite connected component.

But now what if we introduced some local dependencies between the edges? More precisely, suppose each edge still has a probability p of being included in our random subgraph, but its state (present/absent) may depend on the states of edges it shares a vertex with. To what extent can we exploit such local dependencies to delay the appearance of an infinite component?

In this talk I will discuss this question, which first arose in work of Balister, Bollobás and Walters in 2005, and discuss some recent progress on it.

(Joint work with A. Nicholas Day, Robert Hancock and Vincent Pfenninger.)

BERTILLE GRANET - HAMILTON DECOMPOSITIONS OF REGULAR BIPARTITE
TOURNAMENTS

A *regular bipartite tournament* is an orientation of a complete balanced bipartite graph $K_{2n, 2n}$ where every vertex has its in and outdegree both equal to n . In 1981, Jackson conjectured that any regular bipartite tournament can be decomposed into Hamilton cycles. We prove this conjecture for sufficiently large n . Along the way, we also prove several further results, including a conjecture of Liebenau and Pehova on Hamilton decompositions of dense bipartite digraphs.

JAREK GRYTCZUK - TWINS IN COMBINATORIAL STRUCTURES

Twins in a combinatorial structure can be defined as a pair of similar substructures, with respect to various notions of similarity. For instance, in a permutation, twins are formed by two disjoint subsequences having the same relative order. How large twins are guaranteed in every permutation of a fixed length? What if we only take care of the relative order of consecutive terms of subsequences? Or restrict attention to twins occupying specified positions in a permutation, like intervals, for example? One may state similar questions in another fairly general setting of ordered graphs or hypergraphs. For instance, is it true that every planar graph can be edge ordered so that the maximum length of twins forming simple paths is bounded by an absolute constant?

Joint work with Andrzej Dudek and Andrzej Ruciński

ROBERT HANCOCK - BLOWUP RAMSEY NUMBERS

Given graphs G and H , we say $G \xrightarrow{r} H$ if every r -colouring of the edges of G contains a monochromatic copy of H . Let $H[t]$ denote the t -blowup of H . The blowup Ramsey number $B(G \xrightarrow{r} H; t)$ is the minimum n such that $G[n] \xrightarrow{r} H[t]$. Fox, Luo and Wigderson refined an upper bound of Souza, showing that, given G, H and r such that $G \xrightarrow{r} H$, there exist constants $a = a(G, H, r)$ and $b = b(H, r)$ such that for all $t \in \mathbb{N}$, $B(G \xrightarrow{r} H; t) \leq ab^t$. They conjectured that there exist some graphs H for which the constant a depending on G is necessary. We prove this conjecture by showing that the statement is true in the case of H being 3-chromatically connected, which in particular includes triangles. On the other hand, perhaps surprisingly, we show that for forests F , the function $B(G \xrightarrow{r} F; t)$ is independent of G . This is joint work with António Girão.

CARLOS HOPPEN - ON ERDŐS-ROTHSCHILD-TYPE PROBLEMS IN GRAPHS

The problem of characterizing the n -vertex graphs with the maximum number of r -edge-colorings that avoid monochromatic copies of a fixed graph F has been proposed by Erdős and Rothschild in the 1970s. Since then, it has been extended in many ways, having been stated for other combinatorial structures and with different restrictions on the colorings. In this talk, I will describe some recent advances in Erdős-Rothschild-type problems. In particular, I will briefly sketch the proof that, for $r \in \{2, \dots, 26\}$ and large n , the bipartite Turán graph $T_2(n)$ admits the largest number of r -edge colorings avoiding a triangle for which two edges share a color.

BARNABAS JANZER - LARGE HYPERGRAPHS WITHOUT TIGHT CYCLES

An r -uniform tight cycle of length $k > r$ is a hypergraph with vertices v_1, \dots, v_k and edges $\{v_i, v_{i+1}, \dots, v_{i+r-1}\}$ (for all i), with the indices taken modulo k . Sós, and independently

Verstraëte, asked the following question: how many edges can there be in an n -vertex r -uniform hypergraph if it contains no tight cycles of any length? In this talk I will review some known results, and present recent progress on this problem.

OLIVER JANZER - ON THE TURÁN NUMBER OF GRAPHS WITH MINIMUM DEGREE 3

In 1981, Erdős and Simonovits conjectured that for any bipartite graph H we have $\text{ex}(n, H) = O(n^{3/2})$ if and only if H is 2-degenerate. Later, Erdős offered 250 dollars for a proof and 500 dollars for a counterexample. We disprove the conjecture by finding, for any $\varepsilon > 0$, a 3-regular bipartite graph H with $\text{ex}(n, H) = O(n^{4/3+\varepsilon})$.

JAEHOON KIM - HYPERTREES IN STEINER TRIPLE SYSTEMS

A 3-uniform hypertree is a linear 3-graph where every pair of vertices has unique Berge-path between them. Elliott and Rödl conjectured that an n -vertex Steiner triple system contains all possible $(1 - o(1))n$ -vertex hypertrees. We prove this conjecture. This is joint work with Seonghyuk Im, Joonkyung Lee and Abhishek Methuku.

MARCUS KÜHN - CONFLICT-FREE HYPERGRAPH MATCHINGS

A celebrated theorem of Pippenger, and Frankl and Rödl states that every almost-regular, uniform hypergraph \mathcal{H} with small maximum codegree has an almost-perfect matching. We extend this result by obtaining a *conflict-free* matching, where conflicts are encoded via a collection \mathcal{C} of subsets $C \subseteq E(\mathcal{H})$. We say that a matching $\mathcal{M} \subseteq E(\mathcal{H})$ is conflict-free if \mathcal{M} does not contain an element of \mathcal{C} as a subset. Under natural assumptions on \mathcal{C} , we prove that \mathcal{H} has a conflict-free, almost-perfect matching. This has many applications, one of which yields new asymptotic results for so-called “high-girth” Steiner systems. Our main tool is a random greedy algorithm which we call the “conflict-free matching process”.

This is joint work with Stefan Glock, Felix Joos, Jaehoon Kim and Lyuben Lichev.

ANDER LAMAISSON - LOGARITHMIC CONVERGENCE OF PROJECTIVE PLANES

We study the so-called log-convergence of graphs defined by Szegedy. We answer positively his question of whether the sequence of the incidence graphs of projective planes $PG(2, q)$ log-converges, and whether the limit coincides with that of a particular random graph model. Moreover, we show that the sequence is still convergent when q ranges not just over primes, but over prime powers. Joint work with Márton Borbényi, Panna Fekete and Aranka Hrušková.

RICHARD LANG - LOCAL CONDITIONS FOR HYPERGRAPH TILING

The search for local conditions that generate global structure is a major theme in extremal combinatorics. In this talk, we explore the problem at the example of perfect hypergraph tilings and various local conditions such as minimum degrees and quasirandomness. I will give an overview of the progress made so far and then present some new results.

HONG LIU - WHAT DO WE GET FROM CRUX?

I will introduce a notion called crux, which is the size of the smallest dense patch inside a graph. I will show via several examples that this notion captures the ‘essential order’ of a graph from the aspect of graph embeddings. It is also naturally linked to conductance of small sets; one can then use it with isoperimetric inequalities/expansions to obtain improvements on classical results on cycle/subdivision embeddings.

AMANDA MONTEJANO - FEASIBLE EDGE-REPLACEMENTS AND THE FAMILY OF AMOEBAS

The Amoeba Graph-Family was first defined in the context of Ramsey-Turán theory, concerning the study of unavoidable color patterns in 2-edge-colorings of the complete graph where each color class is sufficiently large. Although at a first glance amoebas may seem like a restricted family, it was soon known that, on the contrary, amoebas constitute a wide and rich family of graphs. Roughly speaking, we say that a simple graph G is a *global amoeba*, if given the complete graph K_n for n much larger than the number of vertices of G , and given two copies G_1 and G_2 of G embedded in K_n , the copy G_2 can be reached from G_1 throughout K_n by performing a sequence of edge-replacements such that each edge substitution yields an isomorphic copy of G . A similar concept is that of *local amoeba*, satisfying the same property as global amoebas, but where the host graph K_n has the same number of vertices as G . By means of a group theoretical approach, the dynamics of all of these features can be modeled. In this talk we give formal definitions of the above and present a wide variety of examples. In addition, we present some of the basic and not so basic results that we have obtained. This is a joint work with Yair Caro and Adriana Hansberg.

DÖMÖTÖR PÁLVÖLGYI - EDGE-ORDERED GRAPHS: GEOMETRIC APPLICATIONS

A graph is edge-ordered if there is a linear ordering on its edges. An edge-ordered graph G avoids another edge-ordered graph H , if no subgraph of G has the same order on its edges as H . If H is a path on five vertices with its four edges ordered as 2143, then any H -avoiding graph can have at most $O(n \log n)$ edges and this is sharp. Using this, we give a new proof that the number of unit distances among n planar points in convex position is at most $O(n \log n)$, a bound due to Edelsbrunner and Hajnal, and to Füredi.

If H is a path on five vertices with its four edges ordered as 1324, then also any H -avoiding graph can have at most $O(n \log n)$ edges and this is sharp. Using this, we prove that among

n pairwise disjoint x -monotone red curves and n pairwise disjoint x -monotone blue curves there can be at most $O(n \log n)$ red-blue tangencies, improving the earlier $O(n \log^2 n)$ bound of Pach, Suk, and Tóth, and matching their lower bound construction.

Joint work with Gerbner, Methuku, Nagy, Tardos, and Vizer, and with Keszegh.

OLAF PARCZYK - NEW RAMSEY MULTIPLICITY BOUNDS AND SEARCH HEURISTICS

We present several new improved and tight asymptotic bounds relating both to the minimum number of monochromatic cliques contained in any edge-coloring of a complete graph as well as the minimum number of cliques in graphs with bounded independence number. Most notably, we present the first improvement on the upper bound of the K_4 and K_5 Ramsey multiplicity in more than 20 years and introduce a notion of off-diagonal Ramsey multiplicity. The upper bounds are established through graph and Cayley graph constructions found through computer search heuristics.

This is joint work with Sebastian Pokutta, Christoph Spiegel, and Tibor Szabó.

MAYA STEIN - SEMIDEGREE CONDITIONS FOR ANTITREES

We would like to know for which function $f(k)$ it is true that any oriented graph of minimum semidegree at least $f(k)$ necessarily contains a given oriented path with k edges. For the directed path, $f(k) = k/2$ works, and perhaps this is true for other orientations as well. We show that this is approximately the case for large antidirected paths, and more generally, for large antidirected trees of bounded maximum degree. This is joint work with Camila Zárate.

RAPHAEL STEINER - CHOOSABILITY OF GRAPHS WITH EXCLUDED MINORS

In this talk, I will present a new method for lower-bounding the choosability of graphs with no H -minor, for a fixed graph H . This yields new lower bounds of $(2 - o(1))t$ for the list chromatic number of K_t -minor free graphs and $(1 - o(1))(s + t + \min(s, t))$ for the list chromatic number of $K_{s,t}$ -minor free graphs. These results refute a conjecture of Kawarabayashi and Mohar, and a conjecture of Woodall.

ANDREW TREGLOWN - COMPLETE SUBGRAPHS IN A MULTIPARTITE GRAPH

In 1975 Bollobás, Erdős, and Szemerédi asked the following question: given positive integers n, t, r with $2 \leq t \leq r - 1$, what is the largest minimum degree $\delta(G)$ among all r -partite graphs G with parts of size n and which do not contain a copy of K_{t+1} ? The $r = t + 1$ case has attracted a lot of attention and was fully resolved by Haxell and Szabó, and Szabó and Tardos in 2006. In this talk we investigate the $r > t + 1$ case of the problem, which has remained dormant for over forty years. We resolve the problem exactly in the case when $r \equiv -1 \pmod{t}$, and up to an additive constant for many other cases, including when

$r \geq (3t-1)(t-1)$. Our approach utilizes a connection to the related problem of determining the maximum of the minimum degrees among the family of balanced r -partite rn -vertex graphs of chromatic number at most t . This is joint work with Allan Lo and Yi Zhao.

MYKHAYLO TYOMKYN - MONOCHROMATIC COMPONENTS WITH MANY EDGES

Given an r -edge-coloring of the complete graph K_n , what is the largest number of edges in a monochromatic connected component? This natural question has only recently received the attention it deserves, with work by two disjoint subsets of the authors resolving it for the first two special cases, when $r = 2$ or 3 . Here we introduce a general framework for studying this problem and apply it to fully resolve the $r = 4$ case, showing that such a coloring always yields a monochromatic component with at least $\frac{1}{12} \binom{n}{2}$ edges, where the constant $\frac{1}{12}$ is optimal only when the coloring matches a certain construction of Gyárfás.

Joint work with David Conlon and Sammy Luo.

LEA WEBER - AVOIDABLE ORDER-SIZE PAIRS IN HYPERGRAPHS

For a fixed integer $r \geq 2$, we call a pair (m, f) of integers, $m \geq 1$, $0 \leq f \leq \binom{m}{r}$, *absolutely avoidable* if there is n_0 such that for any pair of integers (n, e) with $n > n_0$ and $0 \leq e \leq \binom{n}{r}$ there is an r -uniform hypergraph on n vertices and e edges that contains no induced sub-hypergraph on m vertices and f edges. Some pairs are clearly not absolutely avoidable, for example $(m, 0)$ is not absolutely avoidable since any sufficiently sparse hypergraph on at least m vertices contains independent sets on m vertices. Here we show that for any $r \geq 3$ and $m \geq m_0$, either the pair $(m, \lfloor \binom{m}{r} / 2 \rfloor)$ or the pair $(m, \lfloor \binom{m}{r} / 2 \rfloor - m - 1)$ is absolutely avoidable.

Next, following the definition of Erdős, Füredi, Rothschild and Sós, we define the *density* of a pair (m, f) as $\sigma_r(m, f) = \limsup_{n \rightarrow \infty} \frac{|\{e: (n, e) \rightarrow (m, f)\}|}{\binom{n}{r}}$. We show that for $r \geq 3$ most pairs (m, f) satisfy $\sigma_r(m, f) = 0$, and that for $m > r$, there exists no pair (m, f) of density 1.

CHRISTIAN WINTER - POSET RAMSEY NUMBER: LARGE BOOLEAN LATTICE VS. FIXED POSET

A Boolean lattice Q_N is a partially order set consisting all subsets of an N -element ground set. For any partially ordered sets (posets) P and Q , the poset Ramsey number $R(P, Q)$ is the least positive integer N such that no matter how the elements of a Boolean lattice Q_N are colored in blue and red, there is either an induced copy of P with all blue elements or an induced copy of Q with all red elements.

In this talk we provide bounds on the poset Ramsey number $R(P, Q_n)$ for a fixed poset P and an n -dimensional Boolean lattice Q_n , as n grows large. We show a sharp jump in behaviour of this number as a function of n depending on whether or not P contains a copy of either a V -shaped poset or its symmetric counterpart.